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Conjugate heat transfer in porous triangular enclosures with thick bottom wall

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Abstract

Purpose – The purpose of this paper is to study the conjugate heat transfer via natural convection and conduction in a triangular enclosure filled with a porous medium.

Design/methodology/approach – Darcy flow model was used to write governing equations with Boussinesq approximation. The transformed governing equations are solved numerically using a finite difference technique. It is assumed that the enclosure consists of a conducting bottom wall of finite thickness, an adiabatic (insulated) vertical wall and a cooled inclined wall.

Findings – Flow patterns, temperature and heat transfer were presented at different dimensionless thickness of the bottom wall, h , from 0.05 to 0.3, different thermal conductivity ratio between solid material and fluid, k, from 0.44 to 283 and Rayleigh numbers, Ra , from 100 to 1000. It is found that both thermal conductivity ratio and thickness of the bottom wall can be used as control parameters for heat transport and flow field.

Originality/value – It is believed that this is the first paper on conduction-natural convection in porous media filled triangular enclosures with thick wall. In the last years, most of the researchers focused on regular geometries such as rectangular or square cavity bounded by thick wall. Keywords Heat transfer, Porous materials, Heat conduction, Convection, Numerical analysis Paper type Research paper

Nomenclature

- $g =$ gravitational acceleration
- $h =$ dimensionless height of the solid wall, h'/H
- h' = height of solid wall
- $H =$ height of triangle or cavity
- K = permeability of the porous medium
- $k =$ thermal conductivity ratio, k_s/k_f
- $L =$ length of the bottom wall, $L = H$
- $n =$ coordinate in normal direction
- $Nu =$ mean Nusselt number
- $Nu_r =$ local Nusselt number
- $Ra =$ Rayleigh number
- $T =$ temperature
- u, v = velocity components in x, y directions
- U, V = dimensionless velocity components in X, Ydirections
- x, y = Cartesian coordinates
- $X, Y =$ non-dimensional coordinates

Greek letters

- α_m $=$ thermal diffusivity of the porous medium
- β = thermal expansion coefficient

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Introduction

Transport of heat through a porous medium has been the subject of various recent studies due to the increasing need for a better understanding of the associated transport processes. This interest stems from the numerous practical applications which can be modeled or can be approximated as transport through porous media such as packed sphere beds, high performance insulation for buildings, chemical catalytic reactors, grain storage, migration of moisture through the air contained in fibrous insulations, heat exchange between soil and atmosphere, sensible heat storage beds and beds of fossil fuels such as oil shale and coal, salt leaching in soils, solar power collectors, electrochemical processes, insulation of nuclear reactors, regenerative heat exchangers and geothermal energy systems and many other areas. Literature concerning convective flow in porous media is abundant. Representative studies in this are may be found in the recent books by Nield and Bejan (2006), Ingham and Pop (2005), Vafai (2005), Pop and Ingham (2001), Ingham et al. (2004) and Bejan et al. (2004).

In conventional heat transfer analyses, it is common practice to consider the temperature or the heat flux at the fluid-wall interface as known *a priori*. The results thus obtained are good only for heat transfer in flows bounded by walls having extremely small thermal resistance, i.e. very high thermal conductivity and/or very small thickness. However, in actual practice, the wall thermal resistance is finite and the thermal conditions at the fluid-wall interface are different from their counterparts imposed at the outer surface of the solid walls. Such type of problems, where heat conduction in the solid is coupled with convective heat transfer in the fluid, is often referred to as conjugate problems (El-Shaarawi et al., 2007). Most of the studies on natural convection in triangular enclosures filled with a fluid-porous media include thin walled triangular cavity by Baytas et al. (2000), Varol et al. (2006, 2007a, b, 2008), Bejan (1979), Poulikakos and Bejan (1983) and Vasseur and Degan (1998). However, studies on the effect of wall conduction in a triangular porous cavity with conducting solid walls of finite thickness are very limited. The results of such studies can be used in building physics, electronic cooling applications etc. Kimura et al. (1997) presented a review study to show different applications of conjugate convection for porous medium. Mbaye *et al.* (1993) studies the natural convection– conduction problem for a rectangular porous cavity to investigate the effect of Rayleigh number and conductivity ratio on thermal and flow field. Baytas et al. (2001) made a numerical analysis to solve conjugate natural convection problem in a square enclosure filled with a fluid-saturated porous medium. Chang and Lin (1994) studied the conductionnatural convection problem for non-Darcian porous media. Mohamad and Rees (2004) studied the conjugate effects on natural convection from a heated vertical flat plate embedded in a porous medium. Recently, Saeid (2007a) made a numerical study to investigate the effect of conduction in one of the vertical wall in conduction-natural convection problem in a porous square enclosure. He observed that either increasing the Rayleigh number and the thermal conductivity ratio or decreasing the thickness of solid bounded wall can increase the average Nusselt number. In the special cases of low Rayleigh number and high conductive walls, the values of the average Nusselt numbers are increasing with the increase of the wall thickness. The problem of conjugate natural

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convection in a vertical porous layer sandwiched by finite thickness walls has been also considered (Saeid, 2007b). He used the Darcy law model in the mathematical formulation for the porous layer and the finite volume method to solve the dimensionless governing equations. He found that as Ra increases the average Nusselt number is increasing with higher slop for the thin walls than that for thick walls. It is worth mentioning to this end that the coupling of wall conduction with natural convection in rectangular enclosures filled with a Newtonian fluid (non-porous media) has been studied by several authors, such as Koutsoheras and Charters (1977), Meyer et al. (1982), Kim and Viskanta (1984, 1985), Kahveci (2007) and Du and Bilgen (1992). These studies were motivated by many engineering applications in electrical, nuclear, thermal storage fields and electronic cooling applications. Heat producing electronic components are often mounted on a printed circuit board above a conducting plate. The heat produced is then transferred, both by conduction through the plate to its two ends and by natural convection in the surrounding fluid to the heat sinks. As a result, the heat removing rate from the electronic components will depend on the coupling of the wall conduction and the fluid convection. This coupling will directly influence the temperature distribution among the components and thus the design of heat removing mechanisms in practical applications (Du and Bilgen, 1992).

The principal aim of the present study is to examine the problem of conductionnatural convection in a triangular enclosure filled with a fluid-saturated porous medium which consists of a conducting bottom wall of finite thickness. Above literature survey clearly shows that conjugate natural convection problems for triangular enclosure are not studied yet. Thus, we believe that the present results are new and very important for some practical applications.

Problem description

The configuration of the right-angle porous triangular enclosure with a conducting thick bottom wall is given in Figure 1(a) along with boundary conditions and coordinates. The thickness of the conducting bottom wall is denoted by h' and it is a solid wall made from different materials. The cavity is heated from the bottom wall with an isothermal heater and temperature of the inclined wall is lower than that of the bottom wall. The vertical wall of the enclosure is insulated.

Mathematical model

In order to write the governing equations for the problem under consideration the following assumptions are made: the properties of the fluid and the porous medium are constant; the cavity walls are impermeable; the Boussinesq approximation and the Darcy law model are valid. With these assumptions, the dimensional governing equations as continuity, momentum and energy can be written as follows

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\tag{1}
$$

$$
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{g\beta K}{v} \frac{\partial T_f}{\partial x}
$$
 (2)

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Figure 1. (a) Geometry and boundary conditions of the problem, (b) Computational domain and grid distribution

$$
u\frac{\partial T_f}{\partial x} + v\frac{\partial T_f}{\partial y} = \alpha_m \left(\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} \right)
$$

and the energy equation for the bottom solid wall is:

$$
\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} = 0\tag{4}
$$

where u and v are the velocity components along x and y axes, T_f is the fluid temperature, g is the acceleration due to gravity, T_s is the temperature of the solid bottom wall, K is the permeability of the porous medium, α_m is the effective thermal diffusivity of the porous medium, β is the thermal expansion coefficient and υ is the kinematic viscosity. Introducing the stream function ψ defined as

654 $u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$ $\frac{\partial \varphi}{\partial x}$ (5)

Equations (1)-(4) can be written in non-dimensional form as

$$
\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -Ra \frac{\partial \theta_f}{\partial X}
$$
(6)

$$
\frac{\partial \Psi}{\partial Y} \frac{\partial \theta_f}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \theta_f}{\partial Y} = \frac{\partial^2 \theta_f}{\partial X^2} + \frac{\partial^2 \theta_f}{\partial Y^2}
$$
(7)

for the fluid-saturated porous medium and

$$
\frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} = 0
$$
\n(8)

for the solid wall, respectively. Here $Ra = g \beta K (T_H - T_C) H/\alpha_m v$ is the Rayleigh number for the porous medium and the non-dimensional quantities are defined as

$$
X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad (U, V) = \frac{(u, v)H}{\alpha_m}, \quad \Psi = \frac{\psi}{\alpha_m}
$$

$$
\theta_f = \frac{T_f - T_C}{T_H - T_C}, \quad \theta_s = \frac{T_s - T_C}{T_H - T_C}
$$
 (9)

The boundary conditions of Equations (6)-(8) are: for all solid boundaries

$$
\Psi = 0 \tag{10a}
$$

on the vertical wall (adiabatic), $0 \le Y \le 1$

$$
\frac{\partial \theta_f}{\partial X} = \frac{\partial \theta_s}{\partial X} = 0
$$
\n(10b)

on the bottom wall (hot), $0 \le X \le 1$

$$
\theta_f = \theta_s = 1 \tag{10c}
$$

on the inclined wall (cold)

$$
\theta_f = \theta_s = 0 \tag{10d}
$$

for the interface between solid and porous media,

$$
k_f \frac{\partial \theta_f}{\partial Y} = k_s \frac{\partial \theta_s}{\partial Y}
$$
 (10e)

In above boundary condition, the total heat flux q_w is assumed to have the same representation as the case of local thermal equilibrium. In other words, it is assumed that both phases have the same temperature and temperature gradient at the wall as indicated by Alazmi and Vafai (2002). Physical quantities of interest in this problem are the local Nusselt number Nu_x at the fluid side and the mean Nusselt number Nu which are given by

$$
Nu_x = \left(-\frac{\partial \theta_f}{\partial Y}\right)_{Y=h}, Nu = \frac{1}{\ell_x} \int_0^{\ell_x} Nu_x dX \tag{11a,b}
$$

and also the Nusselt number Nu_s at the solid part

$$
Nu_s = \left(-\frac{\partial \theta_s}{\partial Y}\right)_{Y=h} \tag{12}
$$

for the interface between the solid bottom wall and porous medium. Based on Equation (10e) the following relation must be satisfied between Nu_x and Nu_s as $Nu_x = (k_s/k_f)Nu_s$ (Saeid, 2007a). Thus, one can determine the value of the Nusselt number for the solid part of the interface.

Numerical technique

Equations (6)-(8) subject to the boundary conditions (10) are integrated numerically using the finite-difference method. Numerical simulations were carried out systematically in order to determine the effect of three main parameters of the problem, namely: Rayleigh number Ra , thermal conductivity ratio k and thickness of the solid bottom wall $h (= h'/H)$ on the flow and heat transfer characteristics. The solution domain, therefore, consists of grid points at which equations are applied. The grid size was selected to be similar to that used by 61x61 for the cavity with uniform grid spacing. Figure 1(b) reveals clearly the grid arrangement. The resulting algebraic equations were solved by successive under relaxation method. The iteration process is terminated under the following condition:

$$
\sum_{i,j} \left| \phi_{i,j}^{m} - \phi_{i,j}^{m-1} \right| \left| \sum_{i,j} \left| \phi_{i,j}^{m} \right| \le 10^{-5} \tag{13}
$$

where *m* denotes the iteration step and ϕ stands for either θ_f , θ_s or Ψ . Due to lack of suitable results in the literature pertaining to the present configuration, the obtained results have been validated against the existing results for a square cavity filled with a porous medium. Thus, the comparison of the present results for the mean Nusselt number Nu , as defined by Equation (11), with those from the open literature has been made for a value of $Ra = 1,000$. Comparison results can be found in our earlier publications as Varol et al. (2006, 2008).

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First, we tested the effects of the Rayleigh number Ra on both the temperatures of the fluid-saturated porous medium and of the solid bottom wall as well as on the flow field in the case of epoxy-water ($k = 0.44$) when $h = 0.1$. Four different solid materials and fluids are chosen according to ratio of thermal conductivity k given in Table I. Thus, Figure 2(a)-(c) shows the effects of Ra on streamlines and isotherms. As can be seen in Figure 2(a) a circle-shaped single rotating cell was obtained and it rotates in clockwise direction with $\Psi_{min} = -2.19$. It can be also seen that the shape of the cell is deformed with increasing Ra and that the flow strength increases. Isotherms show that the temperature distribution is uniform in the solid bottom wall for all values of Ra. For low values of Ra, isotherms are almost parallel to the cold inclined surface due to domination of the conduction mode of heat transfer. However, the isotherms are also distorted with increasing Ra, namely, the convection effects. Figure 3 illustrates the effects of the wall thickness parameter h for $Ra = 500$ and $k = 0.44$ on the temperature distributions and flow fields inside the porous cavity and inside the bottom solid wall. As can be seen the parameter h also affects the fluid and the solid temperatures as well as the flow characteristics. The strength of the circulation of the fluid-saturated porous medium is much higher for a thin bottom wall. This has been found also by Saeid (2007b). Thus, more fluid is heated in the cavity thanks to the conducting solid bottom wall.

The streamlines (on the left) and isotherms (on the right) are shown for some values of the thermal conductivity ratio parameter k in Figure 4(a)-(c) for $Ra = 500$ and $h = 0.2$. To this test, three different materials as epoxy-air ($k = 9.90$), stainless steelwater ($k = 23.8$) and alumina-water ($k = 283$) were used as given in Table I. As can be seen from these figures the higher values of k enhances the flow strength. The reason of this phenomenon is that the temperature gradient near the wall increases with the increase of the parameter k . Thus, much heat transfer from the bottom solid wall to the porous media is obtained for higher values of k. Figure $4(a)$ -(c) also shows that convection effects inside the porous medium become stronger for higher values of k . For alumina-water ($k = 283$), second cell is formed near the right bottom corner of the porous cavity. Similarly, the most part of the solid bottom wall is heated for higher values the thermal conductivity parameter k.

Variation of the mean Nusselt number Nu with Ra , which is calculated from Equation (11), is illustrated in Figure 5 for $k = 0.44$ (epoxy-water) to show the effects of the wall thickness parameter h on heat transfer. The figure shows that for low values of h the heat transfer increases with increasing Ra . This is due to increasing of domination of convection heat transfer inside the porous media. Figure 5 also shows that Nu becomes constant for highest values of the thickness parameter of the solid bottom wall (h = 0.3). Variation of Nu with Ra is shown in Figure 6 for $h = 0.2$ and some values of k. It can be noticed from this figure that higher values of Nu are obtained with increasing the parameter k. Thus, there is a considerable difference between the values of Nu for small and large values of k. This is because the convection inside porous medium increases. For $k = 23.8$, the mean Nusselt number increases with increasing Ra due to increasing of convection dominated heat transfer regime. At low

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values of k the values of Nu are almost constant. Figure 7 is plotted to show the variation of Nu with k for some values of Ra when $h = 0.2$. As can be seen, the values of Nu increases with increasing Ra except for lower values of thermal conductivity ratio parameter k. The values of Nu are increased suddenly for $k > 1$ due to effect of conduction. In this case the bottom solid wall behaves as insulated material. For higher

Variation of the mean Nusselt number as a function of Rayleigh number for different thickness ratio of the bottom wall at epoxywater $(k = 0.44)$

Variation of the mean Nusselt number as a function of Rayleigh number for different conductivity ratios at $h = 0.2$

Variation of the mean Nusselt number as a function of conductivity ratio for different Rayleigh numbers at $h = 0.2$

value of k the convection heat transfer becomes much more important. Variation of Nu with X, the dimensionless distance measured along the lower surface of the bottom solid wall, is presented in Figure 8 for different values of Ra when $h = 0.05$ and $k = 0.44$. It can be noticed that Nu increases with the increase of X. However, when $Ra = 1,000$ it decreases sharply around $X = 0.58$ due to Benard-cell type of flow. A similar result was obtained in earlier studies on non-conjugate natural convection flow in triangular enclosures filled with porous media. Variation of Nu with X is illustrated in Figure 9 for some values of h when $Ra = 1,000$ and $k = 0.44$. It is observed that higher values of Nu are obtained for lower values of h. However, Nu remains almost constant for the values of h considered except the value of $h = 0.05$. Thicker solid wall behaves as insulation material independent of values of k. In this case, convection effects become dominant and Benard-cells were formed. Finally, the effects of thermal conductivity ratio k on the local Nusselt number Nu is displayed in Figure 10 for $Ra = 1,000$ and $h = 0.1$. As this figure suggests Nu remains almost constant and has the lower value for $k = 0.44$, where the solid wall is an insulation material. This finding is supported by the results reported by Saeid (2007b). For higher

Figure 9. Variation of the local Nusselt number for different thickness ratios of the bottom wall at Epoxy-water $(k = 0.44)$ and $Ra = 1,000$

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Figure 10. Variation of the local Nusselt number for different conductivity ratios at $Ra = 1,000$ and $h = 0.1$

values of k the local Nusselt number shows wavy variation due to strong convection. The similar results were shown in earlier studies by Varol et al. (2007a, b).

Conclusions

A numerical study is performed to examine the steady laminar natural convection– conduction in triangular enclosure filled with fluid-saturated porous media with a conducting bottom solid wall for different Rayleigh number, thickness of the bottom wall and thermal conductivity ratio. It is found that the flow strength becomes lower for thin wall or low values of thermal conductivity ratio. It is also found that increasing of thick wall, reduces the mean Nusselt number due to decreasing of temperature difference. For the constant wall thickness and thermal conductivity ratio, Nusselt number increases with increasing of Rayleigh number. For thin wall and high Rayleigh number wavy variation was observed in the local Nusselt number due to increasing of convection effects. Values of thermal conductivity parameter k are effective for $k > 1$.

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